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# A Method for the Description of Minutia Pairs in Epidermal Ridge Patterns 

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#### Abstract

A descriptive method is presented that allows documentation of minutia configurations in epidermal ridge patterns. The method incorporates the basic features relevant to fingerprint comparison: minutia types, orientations, and relative positions. Provision is also made for the ambiguities in minutia type which are an inevitable feature of any fingerprint comparison process. A descriptive method incorporating these features is needed to study systematically the variation of epidermal ridge minutiae and to test the existing hypotheses regarding the frequencies of occurrence of specific minutia configurations.


KEYWORDS: forensic science, fingerprints, comparative analysis, epidermal ridges, pattern analysis

The nature and variability of configurations of epidermal ridge minutiae is of direct forensic science significance in the evaluation and comparison of partial fingerprints. With fingerprints of more extensive area, an argument may be made for identity in the absence of any detailed knowledge of minutia variation; we do have a century of empirical experience upon which to rely. As one considers smaller and smaller portions of the fingerprint, however, the information content decreases, and the need arises for a more systematic study of minutia variability. Whereas the number of matching characteristics accepted for an identification now hovers around eight, the need for systematic study has become acute.

This need has been recognized for some time and has resulted in numerous attempts to provide a quantitative assessment of fingerprint individuality [1-7]. In each case a hypothesis has been formed regarding the variability of the minutiae in fingerprints, and the hypothesis has been offered as justification for conclusions of identity. None of these hypotheses, however, has been tested. To test the hypotheses, we need data regarding the distribution and variation of minutiae. We need to know what does exist in order to test the theoretical predictions. Unfortunately, our current knowledge of minutia variation is insufficient for this purpose because there has been no systematic description of the distribution and variation of epidermal ridge minutiae. A systematic descriptive method for minutiae is required before any rigorous testing of hypotheses regarding the independence of minutia type, orien-

[^0]tation, and relative position. Rules used to describe and document minutia configurations have been developed in connection with the classification process, but curiously they have not been developed in connection with the comparison process.
The present work describes a method for the documentation of minutia configurations. Methods developed in conjunction with quantitative assessments of fingerprint individuality have been reviewed elsewhere [8] and have been found to be deficient in that all of them fail to incorporate the basic features of fingerprint comparison. The ultimate issue is whether the process of fingerprint comparison will find two minutia configurations to correspond. Accordingly, a relevant descriptive method must incorporate the actual features of the comparison process.
The present work begins with a discussion of the fingerprint comparison process and a definition of the essential features to be included in the descriptive method. This is followed by presentation of a method for the description of minutia type, orientation, and relative position. The concept of "neighbor minutiae" is introduced, and it is shown how a fingerprint may be represented as a network of neighbor minutiae. Next, the difficulties encountered with ridge counts of one and zero are considered and it is seen that these must be treated as special cases. Ambiguity of minutia type introduces further complexity, but the descriptive method is shown to be adaptable to these circumstances. The work concludes with a brief discussion of the applications for the descriptive method, along with its limitations.

## The Fingerprint Comparison Process

Fingerprint comparison is a sequential search for corresponding minutiae in two fingerprints; a full description of the process is given by Cowger [9]. To begin the comparison, a common reference point, frequently referred to as a "starting point," is necessary. After the common reference is found, the comparison proceeds by selection of a minutia near the reference point in one of the prints. The minutia type, orientation, and position relative to the reference point are noted. The type is the form of the minutia, typically an ending ridge, fork, or dot. Orientation refers to the direction along the ridge flow; as one proceeds along the ridges from the reference point, does the minutia consume or produce a ridge? Position is established by two measures: (1) the number of ridges separating the minutia from the reference point and (2) the distance between the minutia and the reference point along the ridge flow.

After the type, orientation, and relative position of the minutia has been noted, a similar minutia is sought in a second fingerprint. Two features of this search cannot be compromised: (1) the orientation of the minutia and (2) the number of ridges separating the minutia from the reference point. Some latitude is permitted in the assessment of correspondence in two other features: (1) the relative position of the minutia along the ridges and (2) the minutia type. Minor differences in location of minutiae along the direction of ridge flow must be tolerated because of the possibility of distortion. The skin is pliable and when a fingerprint is made the distances between minutiae may be influenced by the vagaries of the skin-to-surface contact. These same factors may also cause differences in apparent minutia type; a minutia that appears to be an ending ridge in one print may appear to be a fork in another, and vice versa [10-12].
If a corresponding minutia is found in the second fingerprint, the comparison continues by locating additional minutiae in the first fingerprint. Their type, orientation, and position relative to one of the other minutiae are noted. The second fingerprint is then searched for a corresponding minutia. The search for each new corresponding minutia is a test of the hypothesis that the prints were made by the same individual. If an irreconcilable discrepancy is encountered, a conclusion may be made that the two prints do not share a common origin. If correspondence is found, the process may continue until all the information in one of the
fingerprints has been compared, or until a sentient decision is made that an unequivocal identification has been effected.

It can be seen that the minutia pair is the fundamental unit of fingerprint individuality. The pair consists not only of the two minutia events, but also their spatial relationship within the fingerprint pattern.

The comparison process outlined above dictates the specific features that must be incorporated into a comprehensive description of minutia pairs. These features are:
(1) use of ridge count as a measure across the ridge flow,
(2) use of a continuous linear measure along ridges,
(3) description of minutia orientation relative to ridge flow, and
(4) provision for dealing with ambiguities in minutia type.

A method of minutia pair description incorporating these features has been designed and will now be described.

## Description of Minutia Type and Orientation

There are three fundamental minutia types: the fork (bifurcation), the ending ridge, and the dot. Other "compound" forms of minutiae occur when the fundamental types are in close proximity to one another. Various authors have considered the compound minutiae to represent discrete types [7,13-15], but the fundamental forms of fork, ending ridge, and dot are adequate for describing minutiae. Additionally, they avoid the somewhat arbitrary definitions of the compound forms [16,17]. In the method described here, the fundamental minutia forms are assigned code letters $\mathrm{B}, \mathrm{E}$, and D , corresponding to fork (or bifurcation), ending ridge, and dot.

Minutiae may have one of two orientations relative to the ridge flow. Orientations are assigned here by aligning the ridges horizontally. As one follows the ridges from left to right, minutiae that produce new ridges are denoted as positive ( P ) and minutiae that consume ridges are denoted as negative ( N ). Dots have no effect on the number of ridges and have no directional sense. They are assigned a null orientation, designated by the letter $O$.

The orientations assigned to minutiae by this procedure are not absolute because the ridges themselves have twofold rotational symmetry. The orientation of minutiae is reversed if one turns the ridge configuration upside down. In the general case one does not know which of the two rotations is correct and both must be considered. This descriptive method allows assignment of minutia orientations when the ridge contiguration is in either rotation. A simple transformation of the final minutia description provides the rotational isomorph.

Combining the descriptions of minutia type and orientation results in five possible descriptions for individual minutiae:

PB (fork directed to right),
NB (fork directed to left),
PE (ending ridge coming from right),
NE (ending ridge coming from left), and
OD (dot-no orientation).

## Descriptions of Relative Minutia Positions

If one counts the number of ridges along a line connecting two minutiae, the count is generally unambiguous. There is the possibility that the line may pass directly through a third minutia, but conventions to deal with various possibilities of this sort are easily developed. In fingerprint classification, such ridge counts are routinely made between the core and delta regions of fingerprint patterns. Two simple ridge counts between minutiae are illustrated in Fig. 1. Note that it is actually the intervals between the ridges that are counted.


FIG. 1-Simple ridge counts between minutiae. Note that it is actually intervals between ridges that are counted.

For the purpose of describing minutia position, the ridge count cannot be applied so simply. We wish to use the ridge count as an unambiguous measure in the direction perpendicular to the ridges. Consider the ridge configuration in Fig. 2. A ridge count of four is obtained by counting along a perpendicular from Minutia $A$ to the ridge on which Minutia $B$ occurs. Alternatively, a ridge count of five is obtained by counting along a perpendicular from $B$ up to $A$ 's ridge. This difference in ridge counts is a consequence of the ending ridge $C$ which appears in the rectangular region defined by the two ridge counts.

Because of the difference in ridge count when there is an intervening minutia, ridge count is used only in describing the positions of minutiae which are immediate neighbors. Under these circumstances, the ridge count is unaffected by the counting route. When a third minutia appears in the region defined by the two alternative perpendicular ridge counts, the two minutiae are not considered to be neighbors. In Fig. 3, Minutia $A$ has neighbors designated $B$ through $G$. The shaded areas shown are minutia-free, resulting in "nearest neighbor" status. Minutiae designated $H, I$, and $J$ are not neighbors of $A$ as a result of the presence of minutiae in the intervening regions.

Two counting conventions are necessary for the ridge count to be self-consistent. First, when counting to a fork that divides before the perpendicular counting line, the count is made to the nearest branch of the fork. Thus, in Fig. 3 the count from Minutia $A$ to Minutia $G$ is 3 . The second necessary convention is that when counting to an ending ridge which does not extend to the perpendicular counting line, the count is made as if the ridge was extended beyond that point. Thus, in Fig. 3 the count from Minutia $A$ to Minutia $E$ is 3 .

Within a larger minutia configuration, minutia positions are related to one another by counting between neighboring minutiae. An entire minutia configuration may be viewed as a network composed of line segments which join neighboring minutiae, as shown in Fig. 4. The positions of any two minutiae may be unambiguously related by counting along the indicated routes.


FIG. 2-Effect of an intervening minutia on the ridge count. The ridge count from A to the ridge on which B occurs is four. whereas the count from B to the ridge on which A occurs is five. The difference in ridge counts is a consequence of the ending ridge C which appears between A and B .


FIG. 3-Neighbor minutiae. Minutia A has neighbors B through G. Minutiae H, I, and J are not neighbors of A because there are minutiae in the intervening regions.


FIG. 4-A network of minutia pairs. Neighboring minutiae are connected by line segments. We may unambiguously relate the positions of any two minutiae by counting along the indicated routes.

The measurement of distance along ridges is a comparatively simple matter. A convenient unit measure for this distance is the ridge interval. The distance is taken as that between lines drawn perpendicular to the ridges and passing through each minutia. Where the ridges curve these perpendicular "lines" are not parallel, and the distance varies as one proceeds along the ridge count. Either of two conventions may be used under such circumstances. If one is counting from one particular minutia to the other, the distance along the ridge of the second minutia may be used. Alternatively, the distance may be measured along the ridge or ridge interval which is equidistant from the two minutiae; this is illustrated in Fig. 5.


FIG. 5-Measurement of intervening distance. The distance between two minutiae is taken as that between two lines drawn perpendicular to the ridge flow and passing through the minutiae. When the ridge flow shows curvature, the distance may be taken as that along the ridge or ridge interval equidistant from the two minutiae.

## Description of Minutia Pairs

The identity and relationship of a pair of minutiae may be concisely represented by the general formula:

$$
(\mathrm{O} 1)(\mathrm{T} 1)(\mathrm{O} 2)(\mathrm{T} 2)(\mathrm{S})(\mathrm{C})(\mathrm{I})
$$

where O 1 and O 2 are the orientations of the first and second minutiae, T 1 and T 2 are the corresponding minutia types, $S$ is the sign of the ridge count, $C$ is the ridge count, and $I$ is the intervening distance.

Conventions for the coding of minutia type and orientation were presented earlier. Recall that there are three minutia types, B (fork or bifurcation), E (ending ridge), and D (dot). There are also three possible orientations: P (new ridge to right), N (loss of ridge to right), and $O$ (no orientation, applicable only to dots). Permutation of the type and orientation results in five possible descriptions for minutiae: PB, NB, PE, NE, and OD. The sign of the ridge count is taken as positive $(P)$ if the count is upward when counting from Minutia 1 to Minutia 2 and negative $(\mathrm{N})$ if this count is downward. The ridge count is the number of ridge intervals crossed. The intervening distance is positive if Minutia 2 is to the right of Minutia 1 , and negative if Minutia 2 is to the left of Minutia 1.

Examples of this descriptive method are shown in Fig. 6. A given pair may be described by any of four redundant formulas. One of the three additional formulas arises by reversing the direction of counting, and two more arise if the configuration is rotated by $180^{\circ}$. Thus the designation NEPBP2 +1.0 is equivalent to PBNEN2-1.0 if one starts counting from the complementary minutia, and these two designations become PENBN2-1.0 and NBPEP $2+1.0$ if each is rotated $180^{\circ}$. In general, reversing the order of the minutiae changes the sign of the distance and the sign of the ridge count. Rotation through $180^{\circ}$ changes the orientation of each minutia in addition to the sign of the distance and the sign of the ridge count.

## Difficulties with Ridge Counts of One and Zero

The system for minutia pair description is sufficient when the ridge counts are two or more. When minutiae are closer, problems occur that require special treatment. The problems that arise can best be appreciated by examining the full set of possible minutia pairs.

Allowing each of two minutia types ( $B$ or $E$ ), each of two minutia orientations ( $P$ or $N$ ),


NEPBP $2+1.0$
or
PBNEN2-1.0


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NBPBP3-2.0
or
PBNBN3+2.0
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NENEP3+3.0
    Or
NENEN3-30
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FIG. 6-Examples of the descriptive method for minutia pairs presented in this work.
and each of two signs for the intervening distance ( + or - ), sixty-four configurations of minutiae pairs are possible. (We have not included dots because no difficulties arise in their description.) All but ten of these sixty-four possibilities are redundancies for our present purposes; half of them are rotations about the axis parallel to ridge flow, and half again are eliminated because the direction of the ridge count is of no consequence. Of the remaining sixteen configurations, six are rotational isomorphs about the axis perpendicular to the ridges. We are left with ten configurations to examine in detail. These configurations are illustrated in Figs. 7(1) through (10); the minimal ridge count and the two next smallest ridge counts are shown in each figure.

Configuration 1a is assigned a ridge count of zero. This is intuitive and unremarkable. An alternative viewpoint might be that since we need to leave one ridge to count to the other, we have a ridge count of one. The purpose of the ridge count is, however, is to serve as a measure of vertical position, and clearly the two ridges in 1 a are of equal station. Configurations 1 b and $c$ are standard ridge counts of two and three, respectively.

Configuration 2 a is assigned a ridge count of zero: the minutiae are on the same ridge. Configuration $2 b$ has a ridge count of one. This may be contrasted with $1 a$. When the opposing ridges extend past one another, there is a distinction in vertical position and a ridge count of one. Configuration 2 c is unremarkable, with a ridge count of two.

Configurations $3 \mathrm{a}, \mathrm{b}$, and c present no problems. They have ridge counts of one, two, and three, respectively.

Configuration 4 a is assigned a ridge count of zero. This is intuitive and justifiable since the minutiae occupy the same vertical position. Configuration $4 b$ presents the first real difficulty. The two forks appear on the same ridge, and on this basis ought to be assigned a ridge count of zero. Nevertheless, there is a definite vertical positioning of the negative fork above the positive one. Configuration $4 b$ might be called a Z-type based on tracing of the ridge. An


FIG. 7-The ten basic configurations of minutiae. These ten configurations show all possible relationships between minutia types and orientations. The three smallest ridge counts are shown for each case.

S-type, with the positive fork above the negative fork, is fundamentally different. Rotation in the plane of the print will not transform these basic types. To preserve the ridge count and provide the needed sense of vertical positioning, the ridge count from the positive to the negative fork in $4 b$ is assigned a count of $P 0$ (positive zero). Configuration 4 c has a ridge count of one and is no problem.

Configurations $5 \mathrm{a}, \mathrm{b}$, and c are unremarkable. Ridge counts of zero, one, and two, are assigned, respectively. Note now that the ridge count of zero in Configuration 5a has no sign.

To distinguish it from the P 0 and N 0 designations, this ridge count is designated 00 . Counts for Configurations 1a and 2 a would also have this designation.

Configuration 6a employs the "positive zero" convention. The ridge count from the fork positioned to the left to the other fork is P 0 . Configuration 6 b has a ridge count of one, and 6 c has a ridge count of two.
Configurations 7a to c are analogous to Configurations 6a to $\mathrm{c}: 7 \mathrm{a}$ has a ridge count of positive zero, 7 b has a ridge count of one, and 7 c has a ridge count of two.
Configurations 8a, b, and care unremarkable. The counts are one, two, and three, respectively.
Configuration 9 a is of interest. Here the fork spreads around an ending ridge. Such configurations are in fact encountered, but not frequently. The ridge count is zero; both minutiae appear at the same level in the ridge pattern. Configurations 9 b and c are unremarkable.

Configurations 10a to c are analogous to Configurations 5 a to c , with ridge counts of zero, one, and two.

## Description of Larger Minutia Configurations

A minutia configuration may be described using a series of formulas. Consider the four minutiae shown in Fig. 8. Beginning at the uppermost minutia, one description of this configuration is as follows:

> PB
> NEN2+2
> NEN2-3
> PBN3+2

A verbal rendition of this code is as follows: Begin with a fork opening to the right; there is an ending ridge stopping two ridges down and two ridge intervals to the right; from this point there is another ending ridge stopping two ridges down and three ridge intervals to the left; from this point there is a fork opening to the right located three ridges down and two ridge intervals to the right.

The description given is only 1 of 24 that are possible for this minutia configuration. We could begin at any of the other 3 minutiae, and any route among the 4 minutiae is valid because each minutia is a neighbor of the other 3 . Furthermore, $180^{\circ}$ rotation yields another


FIG. 8-A larger configuration of minutiae. Larger configurations of minutiae are described by noting the orientation and type of the first minutia and proceeding from minutia to minutia noting the orientation, type, sign of the ridge count, ridge count, and distance. One possible description for the configuration shown here is: PB. NEN2 +2. NEN2-3, PBN3 +2 .
set of 24 descriptions. For any particular description, the rotational isomorph that follows the same sequence of minutiae is given by reversing the signs of the ridge counts, the orientations, and the distances.

## Using the Descriptive Method for Comparisons

The redundancy of descriptions is not a problem during the comparison process if we proceed in the following manner. Suppose there is a known fingerprint to compare against the configuration in Fig. 8. We may take any of the 24 descriptions as our search criteria. The larger, known fingerprint is first coded by each pair of neighboring minutiae. That is, the entire network of possible routes through the fingerprint is coded. When the print is compared with the configuration of interest, each positive fork within the candidate print would be tested as a possible starting point for the search criteria. If one of the fork's neighbor minutiae meets the criteria for the second minutia in our sequence, one proceeds to that neighboring minutia. Next we search among its neighbors for the third minutia in our sequence. This process continues until the configuration is found, or until the all candidate positive forks in the print have been tested. If the rotational isomorph is to be sought as well, then the search sequence is transformed and each negative fork in the print is tested.

The procedure just outlined closely follows the actual practice of fingerprint comparison. A known fingerprint is tested by searching for compatible minutia sequences. Sometimes an assessment may be made quickly, particularly when ridge patterns or characteristic groups of minutiae are present. In the general case, however, each possible juxtaposition of one print on the other must be tried. Description and comparison by the proposed process exhaustively evaluates the possibility that the two prints have a corresponding configuration.

## Effect of Ambiguities in Minutia Type

A necessary feature of the fingerprint comparison process is allowance for variation in minutia type [10-12]. Vagaries of the printing process may, for example, cause a true fork to appear as an ending ridge, either above or below the ridge bearing the fork (see Fig. 9). Similarly, a true ending ridge may appear as a fork, joining either the ridge above or the ridge below. It should be recognized that quite apart from recording difficulties, the nature of some minutiae may be uncertain on the skin itself.

The term "connective ambiguity" is used here to describe the general phenomenon where one is uncertain of the minutia type. Although connective ambiguity results from uncertainty


FIG. 9-Connective ambiguities. Contingencies of fingerprint recording, deposition, and development cause variation in the appearance of minutiae of the type depicted here. These variations are referred to as "connective ambiguities."
in minutia type, ridge counts are also affected. Figure $10 a$ shows a minutia pair describable by the formula NEPBP3 +1 . The count is from the negative ending ridge, seeking a positive fork three ridges up and one ridge interval to the right. If we allow connective ambiguity in the second minutia, two additional configurations become acceptable. The configuration shown in Fig. 10 c results if the lower branch of the fork is disconnected, and the formula describing the minutia pair becomes NEPEP3+1. In this instance there has been only a change in minutia type. If the upper branch is disconnected, however, the configuration in Fig. $10 b$ results and the formula becomes NEPEP4 +1 . Both the minutia type and the ridge count have changed. Forks will cause the ridge count to increase by one when they break in the direction away from the first minutia and will leave the ridge count unchanged when they break toward the first minutia. Connective ambiguity applied to ridge endings results in the reverse process. Fusion in the direction of the first minutia results in a loss of one from the ridge count, whereas fusion away from the first minutia leaves the ridge count unchanged.

Generalization of these effects is straightforward using the descriptive formula presented here. If one was originally seeking a minutia defined by (O2) (T2) (S) (C) (I), connective ambiguity results in two additional acceptable minutia descriptions. If T 2 is B , these are $(\mathrm{O} 2)(\mathrm{E})(\mathrm{S})(\mathrm{C})(\mathrm{I})$ and $(\mathrm{O} 2)(\mathrm{E})(\mathrm{S})(\mathrm{C}+1)(\mathrm{I})$. If T 2 is E , the two possibilities are $(\mathrm{O} 2)(\mathrm{B})(\mathrm{S})(\mathrm{C})(\mathrm{I})$ and $(\mathrm{O} 2)(\mathrm{B})(\mathrm{S})(\mathrm{C}-1)(\mathrm{I})$. If T 2 is D , there is no issue of connective ambiguity.

When connective ambiguity is allowed for both minutiae in a minutia pair, an additional five configurations become possible. It might appear that six configurations should result because each of the two additional forms for the first minutia can be paired with each of the three forms for the second minutia. One of these six configurations is duplicated, however, leaving five new configurations. The duplication occurs when both minutiae show connective ambiguity in the same direction.

Figure $11 a$ through $e$ show the five additional configurations that result using the NEPBP3 +1 example from Fig. 10. Configurations in Fig. $11 a$ and $b$ result from connective ambiguity in the first minutia only. We have only to change our perspective of the ordering of the two minutiae and apply the generalized transformations indicated above. NBPBP2 +1 results when the ending ridge fuses upward, and NBPBP3 +1 results when the ending ridge fuses downward. The configuration in Fig. 11c is the duplicated one referred to above. It results when either (1) the ending ridge fuses upward and the fork disconnects upward or (2) when the ending ridge fuses downward and the fork disconnects downward. When the connective ambiguities operate in the same direction on both minutiae, the resulting configuration is equivalent. The formula that results in this example is NBPEP3 +1 ; the minutia types have changed, but there is no change in ridge count. The configuration in Fig. 11d, with formula NBPEP4 +1 , results when the ending ridge fuses downward and the fork breaks upward. The minutia types are changed and the ridge count increases by one. The final configuration is shown in Fig. 11e. This results when the ending ridge fuses upward and the


FIG. 10-Effect of connective ambiguity on a minutia pair. When connective ambiguity is allowed in one of the minutiae in a pair, three possible acceptable configurations result.


FIG. 11-Effect of connective ambiguity on a minutia pair. When connective ambiguity is allowed in both minutiae in a pair, five additional acceptable configurations result.
fork breaks downward. The minutia types are changed and the ridge count is decreased by one.

Table 1 summarizes the procedure for deriving the formulas of the seven compatible configurations, given a generalized minutia pair configuration.

## Connective Ambiguities in the Comparison Process

Allowing connective ambiguity for both minutiae in a minutia pair results in eight acceptable configurations. One is the initial description without any connective ambiguity, four have connective ambiguity in one of the minutiae, and three have connective ambiguity in both minutiae. Suppose now that the search criteria consists of a series of minutiae. The initial search is for a compatible minutia pair. Allowing for connective ambiguity, there are eight acceptable configurations. What are the criteria for the next minutia in the sequence? The answer is not simple.

As a first example, suppose we find the minutia types originally sought. These will be referred to as the expected types. The criteria for the third minutia in the sequence is then determined by the standard allowance for connective ambiguity: if the expected type of the third minutia is found it must have the expected ridge count. If the unexpected minutia type is found it may have either the expected ridge count, or it may vary by one ridge (one more ridge if the expected type is a fork, one less ridge if the expected type is an ending ridge).

As a second example, suppose we find the expected type for the first minutia, but the unexpected type for the second minutia. Even though we would accept two possible ridge counts to the second minutia, there is in fact only one ridge count which exists in the print being compared. We know, therefore, what specific connective ambiguity had to occur for the correspondence to be true. For example, if we are seeking an ending ridge at a ridge count of four and we find a fork at a ridge count of three, we know that for the patterns to be compatible the ending ridge would have to be fusing downward. Had we found the fork at a ridge count of four we would know that the ending ridge would have to be fusing upward. This knowledge is important when we seek the third minutia because we must use it to adjust the search criteria. If we know that the ending ridge fuses upward, then any positive ridge counts to a third minutia will be shortened by one, whereas any negative ridge counts will be unaffected. If we know that the ending ridge fuses downward, positive ridge counts to a third minutia will be unaffected and negative ridge counts will be shortened by one.

TABLE 1-Alternative minutia pair descriptions allowed by connective ambiguity.

$$
\begin{aligned}
& \text { Original description: }(\mathrm{O} 1)(\mathrm{T} 1)(\mathrm{O} 2)(\mathrm{T} 2)(\mathrm{S})(\mathrm{C})(\mathrm{I}) \\
& \text { Ambiguity in Minutia } 1 \text { gives the alternative descriptions: } \\
& \qquad \begin{array}{r}
(\mathrm{O} 1)(\overline{\mathrm{T} 1})(\mathrm{O} 2)(\mathrm{T} 2)(\mathrm{S})(\mathrm{C})(\mathrm{I}) \\
(\mathrm{O} 1)(\mathrm{T1})(\mathrm{O} 2)(\mathrm{T} 2)(\mathrm{S})\left(\mathrm{C}^{\prime}\right)(\mathrm{I})
\end{array} \\
& \begin{aligned}
\text { where } \mathrm{C}^{\prime} & =(\mathrm{C}+1) \text { if } \mathrm{T} 1=\text { fork } \\
& =(\mathrm{C}-1) \text { if } \mathrm{T} 1=\text { ending ridge } \\
= & (\mathrm{C}) \text { if } \mathrm{T} 1=\operatorname{dot} \text { (redundant description) }
\end{aligned}
\end{aligned}
$$

Ambiguity in Minutia 2 gives the alternate descriptions:
(O1) (T1) (O2) (T2) (S) (C) (I)
(O1) (T1) (O2) (T2) (S) (C') (I)

$$
\text { where } \begin{aligned}
C^{\prime} & =(C+1) \text { if } \mathrm{T} 2=\text { fork } \\
& =(\mathrm{C}-1) \text { if } \mathrm{T} 2=\text { ending ridge } \\
& =(\mathrm{C}) \text { if } \mathrm{T} 2=\text { dot (redundant description) }
\end{aligned}
$$

Ambiguity in both minutiae gives the alternate descriptions:
(O1) ( $\overline{\mathrm{T1}})(\mathrm{O} 2)(\overline{\mathrm{T} 2})(\mathrm{S})(\mathrm{C})(\mathrm{I})$
(O1) (T1) (O2) (T2) (S) (C +1) (I)
(O1) ( $\overline{\mathrm{T} 1}$ ) (O2) $(\overline{\mathrm{T} 2})(\mathrm{S})(\mathrm{C}-1)(\mathrm{I})$
(All three are redundant if $\mathrm{T} 1=\mathrm{T} 2=$ dot. )

As a final case, suppose that the initial minutia is of the unexpected type. We now are uncertain which connective ambiguity might have resulted in the unexpected type. If we were expecting to start with an ending ridge and we start on a fork, we have no way of knowing whether the fork resulted from the ending ridge fusing upward or downward. Regardless of the direction of the next ridge count, we could be off by one (one less because our initial expected minutia type was an ending ridge). Suppose we count to the second minutia in the sequence. If we find the expected minutia at either the expected count or one less, then we are "in register" and may proceed onward with the comparison using the previously defined methods. If, however, we find the unexpected type at the second minutia, we may or may not be able to infer the direction of the connective ambiguity. Referring back to Fig. 11, we see five alternative configurations of minutia pairs that would be acceptable if we began on the lower point and expected an ending ridge. Figure $11 a$ and $b$ have the expected second minutia type. Figure 11c, $d$, and $e$ are the three possible acceptable minutia configurations with the unexpected second minutia. Three different ridge counts are possible: the originally sought for count, one less, and one more. If the ridge count differs from that originally sought, then we obtain information about the direction of the connective ambiguity. If the ridge count is one less than expected, the fork must have broken toward the original minutia. Had it broken away from the original minutia the only options would be a greater ridge count or an equal ridge count. If the ridge count is one greater than expected, then the fork must have broken away from the original minutia. If the ridge count is as expected, however, then no information is gained.
If we are able to infer the direction of the connective ambiguity, then the number of potential formulas for compatible minutiae at the next step is three. If we are unable to infer the direction of a connective ambiguity, then we must accept five possible descriptions for the next minutia. As soon as any minutia is encountered that is the expected type, or as soon as there is a ridge count that is unexpected, the number of compatible types for subsequent minutiae reduces to three. When one of these events occurs, the print is "in register" with the comparison criteria. Once the print is in register there will be only three possible minutia
descriptions that are acceptable. Until the print is in register, five such descriptions must be allowed.

As a practical matter, a print may be put into register by beginning the comparison at a point that shows no connective ambiguity or following the comparison through to a point where either an expected minutia or an unexpected ridge count is found. The minutia at this point may then be used as the starting point for the comparison.

As one proceeds through the search criteria, going from minutia to minutia within a print, one may keep track of the status of the connective ambiguity by noting if the previous minutia resulted from a connective ambiguity in the positive direction, in the negative direction, or in an unknown direction. The CA status may therefore take on the values $(+)$, $(-)$, or (0). If one locates the expected minutia type, then the CA status is not relevant.

There are thus four possible states when determining the adjustment of the search criteria:
(1) expected type $=$ observed type,
(2) $\mathrm{CA}=+$,
(3) $\mathrm{CA}=-$, and
(4) $\mathrm{CA}=$ unknown.

When the CA status is known, or when the expected type is that which is observed, the search criteria are adjusted so that there are three specific minutia formulas that are compatible with each successive minutia point.

## Further Difficulties with Ridge Counts of One and Zero

The special problems that arise when the ridge count is less than two were discussed previously. Connective ambiguities introduce considerable complexity for these smaller ridge counts.

An algorithm for the adjustment of the search criteria has been developed by noting the effect of connective ambiguity on each possible minutia pair with ridge count less than two. One of three adjustment criteria is used, depending on the particular minutia types, the ridge count, and the sign of the intervening distance along the ridge.

If the next minutia sought is ( $\mathrm{O} 2, \mathrm{~T} 2, \mathrm{~S}, \mathrm{C}, \mathrm{I}$ ), the three search criteria are:
Criterion 1. Search for:

$$
\begin{aligned}
& (\mathrm{O}, \mathrm{~T} 2, \mathrm{~S}, \mathrm{C}, \mathrm{I}) ;(\mathrm{O} 2, \overline{\mathrm{~T} 2}, \mathrm{~S}, \mathrm{C}, \mathrm{I}) ; \text { and }\left(\mathrm{O}, \overline{\mathrm{~T} 2}, \mathrm{~S}, \mathrm{C}^{\prime}, \mathrm{I}\right) \\
& \left(\text { where } \mathrm{C}^{\prime}=\mathrm{C}+1 \text { if } \mathrm{T} 2=\mathrm{B} \text { and } \mathrm{C}-1 \text { if } \mathrm{T} 2=\mathrm{E}\right)
\end{aligned}
$$

Criterion 2. Search for:

$$
\begin{aligned}
& (\mathrm{O} 2, \mathrm{~T} 2, \mathrm{~S}, \mathrm{C}, \mathrm{I}) ;(\mathrm{O} 2, \overline{\mathrm{~T}}, 0,0, \mathrm{I}) ; \text { and }\left(\mathrm{O} 2, \overline{\mathrm{~T}}, \mathrm{~S}, \mathrm{C}^{\prime}, \mathrm{I}\right) \\
& \left(\text { where } \mathrm{C}^{\prime}=\mathrm{C}+1 \text { if } \mathrm{T} 2=\mathrm{B} \text { and } \mathrm{C} \text { if } \mathrm{T}_{2}=\mathrm{E}\right)
\end{aligned}
$$

Criterion 3. Search for:

$$
\begin{aligned}
& (\mathrm{O} 2, \mathrm{~T} 2, \mathrm{~S}, \mathrm{C}, \mathrm{I}) ;\left(\mathrm{O} 2, \overline{\mathrm{~T} 2}, \mathrm{~N}, \mathrm{C}^{\prime}, \mathrm{I}\right) ; \text { and }\left(\mathrm{O} 2, \overline{\mathrm{~T} 2}, \mathrm{P}, \mathrm{C}^{\prime}, \mathrm{I}\right) \\
& \text { (where } \mathrm{C}^{\prime}=1 \text { if } \mathrm{T} 1=\mathrm{D},[\mathrm{~T} 1=\overline{\mathrm{T} 2} \text { and } \mathrm{T} 2=\mathrm{B}] \text {, or } \\
& {[\mathrm{T} 1=\mathrm{T} 2 \text { and } \mathrm{O} 2=\operatorname{sign} \text { of } \mathrm{I}] ;} \\
& \text { and } \mathrm{C}^{\prime}=0 \text { if }[\mathrm{T} 1=\overline{\mathrm{T} 2} \text { and } \mathrm{T} 2=\mathrm{E}] \text { or } \\
& [\mathrm{T} 1=\mathrm{T} 2 \text { and } \mathrm{O} 2=- \text { sign of } \mathrm{I}])
\end{aligned}
$$

Criterion 1 is used whenever the ridge count is greater than one and whenever else Criteria 2 or $\mathbf{3}$ do not apply. Criterion 3 is used when $S=0$. Criterion 2 is used if $C=0$ or $C=1$ and one of the following holds:

- $\mathrm{T} 1=\mathrm{T} 2, \mathrm{O} 1=\mathrm{O} 2$, and either $(\mathrm{C}=0$ and $\mathrm{T} 2=\mathrm{B})$ or $(\mathrm{C}=1$ and $\mathrm{T} 2=\mathrm{E})$
- $\mathrm{T} 1=\overline{\mathrm{T} 2}$ and $\mathrm{C}=0$
- $\mathrm{O} 1=\mathrm{P}, \mathrm{T} 1=\mathrm{B}, \mathrm{O} 2=\mathrm{N}, \mathrm{T} 2=\mathrm{E}, \mathrm{C}=1$ and $\mathrm{I}<0$
- $\mathrm{O} 1=\mathrm{N}, \mathrm{T} 1=\mathrm{E}, \mathrm{O} 2=\mathrm{P}, \mathrm{T} 2=\mathrm{B}, \mathrm{C}=1$ and $\mathrm{I}>0$
- $\mathrm{T} 1=\mathrm{D}, \mathrm{T} 2=\mathrm{B}$ and $\mathrm{O} 2=$ sign of I
- $\mathrm{T} 1=\mathrm{D}, \mathrm{T} 2=\mathrm{E}$ and $\mathrm{O} 2=-\operatorname{sign}$ of I

Criteria 2 and 3 are necessary because of the 00 possibility for the ridge count. Criterion 3 is used when the 00 count is expected and Criterion 2 is used when the 00 count is one of the types that would be formed by connective ambiguities. Examples of the application of Criteria B and C are shown in Figs. 12 and 13.

## Discussion

The descriptive method presented here was designed to allow a relevant systematic study of minutiae in epidermal ridge patterns. The authors have successfully used the method to describe the distribution of fingerprint minutiae on the distal portion of male thumbprints [18]. This description permitted the first rigorous testing of hypotheses regarding the independence of minutia type, orientation, and relative position.

The descriptive method presented here can also be used efficiently to record observations when examining and comparing fingerprints; the nature of the minutiae and the particular sequence of comparison is easily documented. When used for this purpose, however, the


FIG. 12-Examples of the use of Criterion 2. Criterion 2 is used to determine alternative acceptable configurations when connective ambiguity results in a configuration where the two minutiae would appear on the same ridge.


FIG. 13-Examples of the use of Criterion 3. Criterion 3 is used to determine alternative acceptable configurations when the initial configuration has minutiae on the same ridge.
method can only be applied when the sequence of minutia pairs defines a convex region. A convex region is one in which a line segment joining any two points within the region is wholly contained within the region. Consider, for example, the ridge and minutia configuration shown in Fig. 14. The region defined by the ridges is not convex; the shaded area shows no ridge structure, and a line from Minutia $B$ to Minutia $C$ is not wholly contained in the region. Note that only Minutiae $A$ and $B$ are neighbors. Minutia $C$ cannot be reached within the constraints of the method. Nevertheless, there is a clear spatial relationship between $B$ and $C$ which is established by counting around the troublesome region. The descriptive method presented here cannot be directly applied to such regions; as a practical matter, however, one may define some arbitrary point along an intermediate ridge and measure the positions of the two minutiae relative to this reference point.

A third possible use for this descriptive method is as an automated comparison algorithm. The method has at least two weaknesses when used for this purpose. First, there is no provision for the comparison of regions where minutiae are absent. Suppose the configuration in Fig. 15 is to be sought. The descriptive method for this configuration documents only the shaded regions in the figure. The outer, minutia-free regions cannot be described by the method, yet these would be an important part of any comparison [7].
The second weakness of this method as a comparison algorithm results from its strictly topological nature. We have incorporated the ridge interval as our only scale parameter. Accordingly, the actual ridge breadth and absolute size of the ridge configuration are not available for comparison. Whereas these features are important to include in actual fingerprint comparison, their introduction is avoided here because the purpose of this descriptive method is to study the patterns themselves.


FIG. 14-A minutia configuration which is not convex. Minutia C cannot be reached within the constraints of the method described here because it is not a neighbor of either Minutia A or Minutia B. Nevertheless a clear spacial relationship exists between Minutia C and the other two minutiae.


FIG. 15-Area documented by the descriptive method. Only the shaded areas in this figure are documented by the descriptive method presented here. This includes the minutiae themselves and the regions between successive minutiae. The outer regions without minutiae are not included in the description, although they would contribute to a fingerprint comparison.

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